

Stratifying process of a fluid in an enclosure with a time-varying vertical through-flow

JAE MIN HYUN† and KI MOON IN

Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology,
P.O. Box 150, Cheong Ryang, Seoul, Korea

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Abstract—Extending an earlier idea by Walin and Rahm, an efficient method of producing a stably stratified fluid system in a container is explored. By pumping a small vertically-upward mass flux through the system, it has been shown that the time to reach the final state is greatly reduced below the diffusive time-scale. The effect of a small pulsating component $\epsilon \sin(\omega t)$ embedded in the through-flux is investigated. Time-dependent solutions to the governing Navier–Stokes equations formulated for a vertically-mounted circular cylinder have been obtained. The overall time-scale for the global motion is still characterized by the usual convective time-scale. However, the velocity fluctuations in the interior core in the final state demonstrate marked variations with ω . The velocity fluctuations are magnified when the imposed pulsating frequency ω reaches a value close to the characteristic buoyancy frequency of the system. This behaviour is insensitive to the variations of the amplitude of the pulsating part; this suggests a resonance phenomenon in the system. Detailed descriptions are provided for the flow patterns inside the cylinder when the amplitude of the pulsating component is appreciable.

1. INTRODUCTION

PRODUCING a fluid system of desired stable thermal stratification is of importance in the laboratory as well as in industrial applications. One obvious way is to confine a fluid between two horizontal, thermally-conducting plates and to impose a vertically stabilizing temperature differential ΔT (hot above cold). If the non-horizontal sidewalls, separating the interior fluid from the environment, are completely insulating, conduction is the only heat transfer mode. At the steady state, a linearly-stratified motionless fluid layer will be obtained. However, this method of producing a stratified fluid, relying solely on heat conduction, is inefficient since the overall transient process takes place over the diffusive time-scale $O(h^2/\kappa)$, h being the thickness of the fluid layer, and κ the thermal diffusivity of the fluid.

In a series of recent papers, Walin and Rahm [1–4] proposed an improved technique of stratifying a contained fluid. The flow model of Walin and Rahm has, among others, two notable features. First, they allowed the non-horizontal boundaries to be imperfect insulators. Second, a small vertically-upward mass flux M , the temperature at the bottom inlet of which was controlled, was pumped through the porous horizontal boundaries. The key ingredient of this model is that, by introducing the above-stated two features, convection becomes a dominant factor in controlling the flow and temperature fields in the interior core. Consequently, the adjustment time is

given by the convective time-scale, which is an order of magnitude smaller than the diffusive time-scale. It has been ascertained that, when the overall convective adjustments have been completed, the interior core is substantially motionless with a nearly linear thermal stratification.

The practical utility of this model has been validated by several preliminary laboratory experiments [1–4]. This flow model offers a significant new idea for obtaining stratified fluids in the laboratory. Furthermore, this model helps us to form a proper perspective of the nature of the physical mechanisms involved in the stratifying process of a fluid in a container. The determination and/or control of the stratification in the interior of a contained fluid is of vital concern in the design of a thermal energy storage system. In particular, the time-dependent process of stratification build-up or decay constitutes an issue of great importance to the operation of a thermocline energy storage tank [5]. Rahm [4], further delineating the theoretical foundation of refs. [1–3], presented an analytical account of the transient evolution as well as the steady state of a stratified fluid in a vertically-mounted circular cylinder when a vertical through-flow M was abruptly applied to the system. The through-flow M was considered to be constant in time and it was uniformly distributed over the entire area of the horizontal endwall disks. For similar problems, Hyun and Hyun [6] acquired numerical solutions to the governing unsteady Navier–Stokes equations. These studies exhibited that, for the parameter values pertinent to realistic thermal systems in the laboratory, the fluid behaviour in the bulk of the container was dominated by convective activities. Detailed descriptions of the

† Author to whom correspondence should be addressed.

$$\frac{\partial T}{\partial r} = s(T_e - T) \quad \text{at } r = a \quad (1)$$

where T_e is the temperature of the environment and is assumed to be intermediate between T_b and T_t . In this model, the thermal conductance at the sidewall is represented by s . Physically, $s = k_w/(kd)$, where k_w and k denote the thermal conductivity of the sidewall material and of the fluid, respectively, and d the thickness of the sidewall.

The fluid motions are governed by the time-dependent Navier–Stokes equations under the Boussinesq-fluid assumptions. Formulated on a cylindrical frame (r, z) with respective velocity components (u, w) , these equations can be expressed as

$$\frac{\partial u}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r}(ru^2) - \frac{\partial}{\partial z}(uw) - \frac{1}{\rho_r} \frac{\partial p}{\partial r} + v \left(\nabla^2 u - \frac{u}{r^2} \right) \quad (2)$$

$$\frac{\partial w}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r}(ruw) - \frac{\partial}{\partial z}(w^2) - \frac{1}{\rho_r} \frac{\partial p}{\partial z} + \alpha g(T - T_e) + v \nabla^2 w \quad (3)$$

$$\frac{\partial T}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r}(ruT) - \frac{\partial}{\partial z}(wT) + \kappa \nabla^2 T \quad (4)$$

$$\frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \quad (5)$$

where

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.$$

In the above, p denotes the reduced pressure, T the temperature, and ρ the density. The relevant fluid properties are ν , kinematic viscosity; κ , thermal diffusivity; α , coefficient of volumetric expansion. The equation of state is $\rho = \rho_r[1 - \alpha(T - T_r)]$, in which subscript r represents the reference values at $z = 0$.

In summary, the associated initial and boundary conditions can be written as

$$u = w = 0, T = T_t(z) \quad \text{at } t = 0 \quad (6)$$

$$u = 0, w = \frac{M}{\pi a^2}, T = T_b \quad \text{at } z = 0 \quad (7)$$

$$u = 0, \frac{\partial w}{\partial z} = 0, T = T_t \quad \text{at } z = h \quad (8)$$

$$u = w = 0, \frac{\partial T}{\partial r} = s(T_e - T) \quad \text{at } r = a. \quad (9)$$

In conformity with the problem statement, the through-flux M is now given a form

$$M = M_0(1 + \varepsilon \sin(\omega t)). \quad (10)$$

The above system of equations is solved by using a modified version of the finite-difference numerical

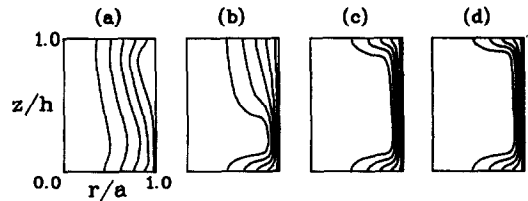


FIG. 1. Representative contour maps of the stream function ψ , shown in units of $w_0 a^2/2$, for $\varepsilon = 0$, ψ is defined such that $u = (1/r)(\partial\psi/\partial z)$, $w = -(1/r)(\partial\psi/\partial r)$ (contour interval $\Delta\psi = 0.19$): (a) at $t/\tau = 0.08$, $\psi_{\max} = 0$, $\psi_{\min} = -1.11$; (b) at $t/\tau = 0.66$, $\psi_{\max} = 0$, $\psi_{\min} = -1.00$; (c) at $t/\tau = 1.29$, $\psi_{\max} = 0.05$, $\psi_{\min} = -1.00$; (d) at $t/\tau = 2.33$, $\psi_{\max} = 0.06$, $\psi_{\min} = -1.00$.

model developed by Warn-Varnas *et al.* [7]. The specifics of the numerical procedures have been documented in refs. [6, 7], and they will not be repeated here.

3. NUMERICAL RESULTS AND DISCUSSION

A large number of numerical runs were performed using the same values of the external parameters as in ref. [6]: $a = 3.0$ cm, $h = 7.0$ cm, $\alpha = 2.86 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$, $\nu = \kappa = 8.3 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$, $\Delta T = 10^\circ\text{C}$, $s = 1.5 \text{ cm}^{-1}$, $M_0 = 1.65 \text{ cm}^3 \text{ s}^{-1}$. These give the significant non-dimensional parameters as follows: the system Rayleigh number $Ra = [\alpha g \Delta T h^3 / \nu \kappa] = 1.40 \times 10^7$, the Prandtl number $Pr = [\nu / \kappa] = 1.0$, $Nh^2/\nu = 3.7 \times 10^3$, where the nominal system Brunt–Väisälä frequency N is defined as $(\alpha g \Delta T / h)^{1/2} = 0.633 \text{ rad s}^{-1}$. Note that these values are consistent with the basic constraints inherent to the model (see refs. [1–4]), i.e. $Ra \gg 1$, $Pr \simeq O(1)$, $Nh^2/\nu \gg 1$, $sh \ll (\kappa / Nh^2)^{-1/2}$. In line with the previous studies [4, 6], the temperature of the environment was set as $T_e = (T_t + T_b)/2$. As a specific example for numerical calculations, the initial vertical temperature profile was set as $T_t(z) = T_b + (0.3 + 0.6z/h)\Delta T$ (see ref. [4]).

In the ensuing plots, the temperature is made dimensionless by $\theta = (T - T_b)/(T_t - T_b)$, and the velocity components are nondimensionalized by using the reference speed of the through-flux, $w_0 \equiv M_0/\pi a^2$. The time is referred to the overall convective time-scale $\tau \equiv a/(2\kappa s)$ (see refs. [4, 6]).

We shall first briefly review the gross characteristics of flow and thermal fields for the case of a steady through-flux, i.e. $\varepsilon = 0$ in equation (10). The details for this case were elaborated previously [4, 6]; only the exemplary highlights are summarized here. Figure 1 depicts the evolution of the flow field. As was pointed out in refs. [4, 6], the global adjustment is substantially accomplished over a time-scale given by the above-stated τ . It is apparent in Fig. 1 that, at moderate times and afterwards, the bulk of the meridional fluid transport in the flow field is carried via the sidewall boundary layer. Figure 2 shows the representative profiles of the temperature and vertical velocity. The presence of a thermal boundary layer

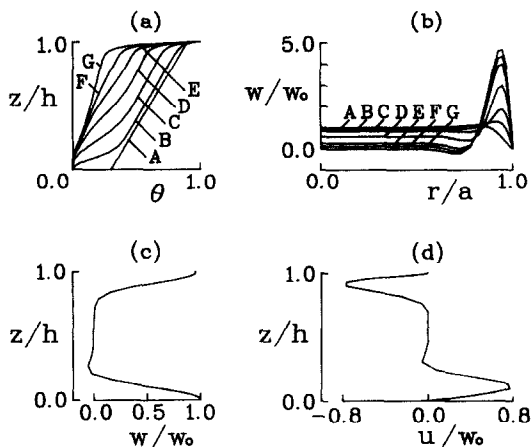


FIG. 2. Temperature and velocity profiles for $\varepsilon = 0$. (a) Vertical temperature profiles along $r/a = 0.5$. (Times, t/τ , are: A, 0; B, 0.08; C, 0.25; D, 0.46; E, 0.67; F, 0.88; and G, 2.33.) (b) Radial profiles of the vertical velocity along $z/h = 0.5$. (Times, t/τ , are: A, 0.08; B, 0.25; C, 0.46; D, 0.67; E, 0.88; F, 1.08; and G, 2.33.) (c) Vertical profile of w along $r/a = 0.5$ at the steady state. (d) Vertical profile of u along $r/a = 0.5$ at the steady state.

near the top endwall disk is discernible in Fig. 2(a). Inspection of Figs. 1, 2(b) and (d) clearly points to the conclusion that, in the steady state, the interior core is substantially stagnant with a nearly linear thermal stratification. These findings, documented earlier in refs. [4, 6], are consistent with the predictions of refs. [1–3].

We shall now scrutinize the effect of the presence of a small-amplitude pulsating component $\varepsilon \sin(\omega t)$ in the through-flux M of equation (10). Accordingly, numerical solutions to the governing time-dependent Navier–Stokes equations have been obtained for a total of 25 runs using $\varepsilon = 0.1$ and ω varying in the range of $(0.2\text{--}15.0)N$.

The overall qualitative patterns of flow evolution when a pulsating component is present are generally similar to those displayed in Fig. 1 for $\varepsilon = 0$. However, even for a pulsating component of such a small amplitude ($\varepsilon = 0.1$), the differences are appreciable in the magnitudes of the interior velocities. Figure 3 presents an illustrative comparison of the vertical velocities in the interior core for the two cases. For the case of a steady through-flux ($\varepsilon = 0$), the oscillatory behaviour is noticed at early times (see Fig. 3(a)). This oscillation is attributable to the internal gravity waves which are

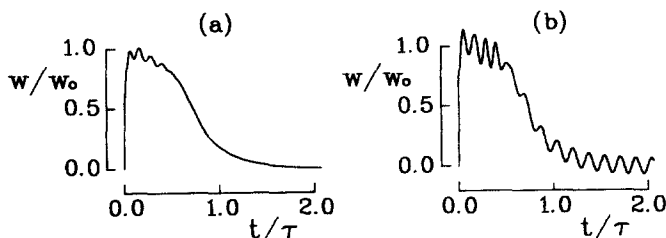


FIG. 3. Time history of w at $r/a = 0.5$, $z/h = 0.5$: (a) $\varepsilon = 0$; (b) $\varepsilon = 0.1$, $\omega/N = 0.5$.

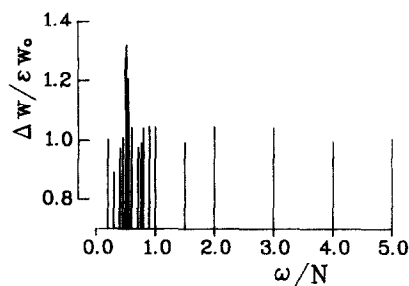


FIG. 4. The velocity fluctuation Δw in the final state vs ω , $\varepsilon = 0.1$.

excited by the impulsive start of the through-flux. These oscillations are damped out after several cycles. It should be remarked that the existence of the oscillatory approach to the steady state in the stratifying process has been intensely discussed lately (e.g. refs. [8–12]). For the case of a non-zero ε (see Fig. 3(b)), a different picture emerges. When the interior velocity is time averaged over a period of the pulsating cycle, the global pattern thus obtained is qualitatively similar to that of $\varepsilon = 0$. However, the time dependency of the velocity itself is manifested; even at large times, the velocity oscillates persistently due to the externally-applied forcing component $\varepsilon \sin(\omega t)$. As stated earlier, this oscillation of velocity at the final state represents the possible mixing of the fluid components in the interior; this will lead to the weakening of the thermocline and/or stratification degradation. Therefore, the central issue here is the strength of the velocity fluctuation due to the presence of $\varepsilon \sin(\omega t)$.

Figure 4 exhibits the variation of the amplitude of the interior velocity fluctuation Δw in the final state vs the frequency of the pulsating component ω . The amplitude of the pulsation was fixed at $\varepsilon = 0.1$. It is evident in the figure that the velocity fluctuation peaks near $\omega/N = 0.5$; this behaviour is clearly suggestive of resonance. Notice that the system Brunt–Väisälä frequency N in the figure is the nominal Brunt–Väisälä frequency, based on the externally-specified temperature differential ΔT . We recall that the maximum temperature difference in the initial state $T_i(z)$ was $0.6\Delta T$. In the final state, Fig. 2(a) shows that the temperature differential in the interior core region, excluding the top thermal boundary layer, is close to $0.30\Delta T$. Consequently, if a more relevant temperature differential is used for the calculation of N , the effec-

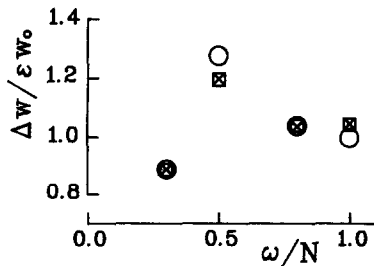


FIG. 5. The velocity fluctuation Δw in the final state vs. ω . (Conditions are: \times , $\varepsilon = 0.02$; O , $\varepsilon = 0.1$; \square , $\varepsilon = 0.5$.)

tive system buoyancy frequency N_e is close to $0.5N$. The results in Fig. 4 are clearly supportive of the argument for resonance, i.e. the interior velocity fluctuations are amplified when the condition $\omega \simeq N_e$ is nearly satisfied.

In summary, the above findings illustrate the possibility of the magnifications of the interior motions. As commented earlier, in the geophysical fluid laboratory, it is often needed to secure, by an efficient means, a stratified motionless fluid system. The present study serves a notice that a small time-dependent component embedded in the through-flux can cause considerable interior motions if certain resonance conditions are fulfilled.

The results demonstrated so far were acquired with the amplitude of the pulsating part $\varepsilon = 0.1$. It is now of interest to examine the effect of the magnitude of ε on the resonance condition. For this purpose, extensive calculations were made using three different values of ε , i.e. 0.02, 0.1, 0.5. Figure 5 displays the variations of the velocity fluctuations in the final state vs the imposed frequency ω/N . It is clear in Fig. 5 that the magnification of the interior velocity fluctuations takes place near the same value of ω/N , and this trend is quite independent of ε . The observations in Fig. 5 further strengthen the previous argument that the interior flow fluctuations are amplified principally as a result of resonance in the system.

It is also noteworthy that, for the present computational results, the overall adjustment time for the interior flow to attain the final state has been found to be quite insensitive to the variations in ε and ω . This implies that, even when a pulsating component is embedded in the imposed through-flux, the characteristic convective time-scales for the global

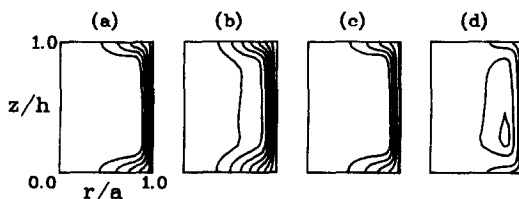


FIG. 6. Contour maps of ψ over a pulsating cycle in the final state, $\varepsilon = 0.5$ (contour interval $\Delta\psi = 0.19$): (a) $\omega t = 0$, $\psi_{\max} = 0.05$, $\psi_{\min} = -1.00$; (b) $\omega t = \pi/2$, $\psi_{\max} = 0$, $\psi_{\min} = -1.50$; (c) $\omega t = \pi$, $\psi_{\max} = 0.06$, $\psi_{\min} = -1.00$; (d) $\omega t = 3\pi/2$, $\psi_{\max} = 0.43$, $\psi_{\min} = -0.50$.

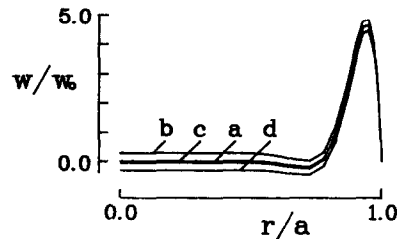


FIG. 7. Profiles of the vertical velocity at $z/h = 0.5$ over a pulsating cycle in the final state, $\varepsilon = 0.5$: (a) $\omega t = 0$; (b) $\omega t = \pi/2$; (c) $\omega t = \pi$; (d) $\omega t = 3\pi/2$.

flow field are given in a manner similar to the case of a steady through-flux.

In order to form a proper perspective of the flow behaviour when the magnitude of the pulsating part is significant, the stream patterns at several phases over a single period in the final state are shown in Fig. 6 for the case of $\varepsilon = 0.5$. As is evident in Figs. 6(a)–(d), the velocity of the through-flux at the bottom endwall disk executes a sinusoidal variation over a full cycle. For most of the time, the dominant portion of the vertical fluid transport is accommodated by the sidewall boundary layer. However, since the amplitude of the pulsating part is appreciable in this case, the gross flow patterns do show considerable variations with the phase angle ωt . Notice that when $\omega t = \pi/2$ (see Fig. 6(b)), because of a vigorous pumping at the inlet ($w = 1.5w_0$), the vertical flow in the main interior core is nonnegligible.

In conjunction with the plots of the stream functions shown in Fig. 6, it is of interest to examine in more detail the buoyancy-layer response to the periodic forcing. Figure 7 illustrates the velocity profiles at the mid-depth $z/h = 0.5$ over a forcing period. Concentration of the vertical motions in the boundary-layer region adjacent to the sidewall is apparent.

4. CONCLUSION

The effect of the pulsating part included in the through-flux M has been analysed. The gross patterns of the global transient flow behaviour remain qualitatively unchanged by the inclusion of a small unsteadiness in the through-flux. However, in the final state, the velocity fluctuations in the interior core are magnified when certain resonance conditions are met, i.e. ω/N takes a certain value. This behaviour is quite insensitive to the magnitude of ε . This observation gives credence to the argument that the amplification of the velocity fluctuations in the interior core is mainly caused by the resonance in the system.

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PROCEDE DE STRATIFICATION D'UN FLUIDE DANS UNE ENCEINTE AVEC UN DEBIT VERTICAL DE TRAVERSEE FONCTION DU TEMPS

Résumé—On explore une idée de Walin et Rahm, une méthode efficace de production d'un système stable de fluide stratifié. En pompant verticalement vers le haut un petit débit à travers le système, il a été montré que le temps pour atteindre l'état final est fortement réduit au dessous de l'échelle de temps de diffusion. L'effet d'une petite composante pulsatoire $\varepsilon \sin(\omega t)$ noyée dans le débit est étudié. Des solutions temporelles des équations de Navier–Stokes ont été obtenues pour un cylindre circulaire vertical. L'échelle globale de temps du mouvement général est caractérisé par l'échelle de temps classique de diffusion. Néanmoins les fluctuations de vitesse dans le noyau interne pendant l'état final montrent des variations sensibles avec ω . Les fluctuations de vitesse sont amplifiées quand la fréquence imposée ω atteint une valeur proche de la fréquence de flottement caractéristique du système. Ce comportement est insensible aux variations de l'amplitude de la partie pulsatoire : cela suggère un phénomène de résonance dans le système. Des descriptions détaillées sont fournies pour les configurations de l'écoulement dans le cylindre lorsque l'amplitude de pulsation est appréciable.

SCHICHTUNG IN EINER FLÜSSIGKEIT IN EINEM BEHÄLTER MIT ZEITLICH VERÄNDERLICHER SENKRECHTER DURCHSTRÖMUNG

Zusammenfassung—Es wird eine effiziente Methode zur Erzeugung stabil geschichteter Flüssigkeitssysteme in einem Behälter untersucht; dies stellt eine Erweiterung einer früheren Idee von Walin und Rahm dar. Durch Hindurchpumpen eines aufwärtsgerichteten Massenstroms durch das System wird gezeigt, daß die Zeit, nach welcher der Endzustand erreicht ist, stark reduziert wird und unterhalb der Zeitkonstanten für Diffusion liegt. Der Effekt einer kleinen pulsierenden Komponente $\varepsilon \sin(\omega t)$ bei der Durchströmung wird untersucht. Als Ergebnis resultieren zeitabhängige Lösungen der Navier–Stokes Gleichungen für einen vertikalen Kreiszylinder. Die Zeitkonstante für den globalen Bewegungsvorgang wird durch die gewöhnliche konvektive Zeitkonstante gekennzeichnet. Im Endzustand zeigen die Schwankungen der Strömungsgeschwindigkeit im Innern jedoch markante Veränderungen mit ω . Diese Schwankungen vergrößern sich, wenn die aufgebrachte Pulsfrequenz einen Wert nahe der charakteristischen Auftriebsfrequenz des Systems erreicht. Dieses Verhalten ist von der Amplitude des pulsierenden Teils unabhängig, was auf ein Resonanzphänomen im System hindeutet. Die Strömungsverteilung im Zylinder bei beträchtlicher Amplitude der pulsierenden Komponente wird detailliert beschrieben.

ПРОЦЕСС СТРАТИФИКАЦИИ ЖИДКОСТИ В ПОЛОСТИ С ИЗМЕНЯЮЩИМСЯ ВО ВРЕМЕНИ ВЕРТИКАЛЬНЫМ СКВОЗНЫМ ПОТОКОМ

Аннотация—На основе предложенной ранее Уолином и Рахом идеи исследуется эффективный метод создания устойчивой стратифицированной жидкой системы в контейнере. Показано, что при прокачке через систему вверх по вертикали массового потока небольшой интенсивности время достижения конечного состояния становится значительно меньше характерного времени диффузии. Изучено влияние наложения на прокачиваемый поток пульсирующей компоненты небольшой амплитуды вида $\varepsilon \sin(\omega t)$. Найдены нестационарные решения уравнений Навье–Стокса для вертикального кругового цилиндра. Полное характерное время для глобального движения остается соизмеримым с обычным временем конвективного перемешивания. Однако флуктуации скорости во внутреннем ядре в конечном состоянии существенно зависят от величины ω . При стремлении частоты наложенной пульсирующей компоненты к характерной частоте плавуемости системы флуктуации скорости возрастают. Поскольку описанное поведение не зависит от амплитуды пульсирующей компоненты, это указывает на наличие резонанса в системе. Приведены подробные картины течения внутри цилиндра при большой амплитуде пульсирующей компоненты.